Chapter 4 – Revised Exercises

Exercise 4.1 Case 1

1. The purpose of this exercise, using the Excel file "Chapter 4 – Case 1", is to explain how the optimal harvest path can be determined in the Case 1 scenario where harvest is a function of stock size and there are decreasing returns to effort.

On the worksheet entitled Basics, the open access dynamics of this case are explained. This is analogous to the analysis in previous exercises and is explained in more detail in section 2 below.

In the text, the problem of finding the optimal harvesting path that will maximize the NPV of harvest is broken down in to two steps.

1. Where to go?
2. How to get there?

There is a worksheet devoted to each of these questions. The first question is relatively easy to answer. It is just a matter of solving the Golden Rule Equation for the terminal optimal stock size given the values of the parameters. Call the solution of the golden rule X_{DMEY}.

The second question of finding a harvest path from a given initial stock size to X_{DMEY} is somewhat more difficult. But it can be accomplished by using a stock projection model which shows how stock size will changes for a given harvest pattern. The model can be specified for any number of years, but using a four year example, stock size will change as follows for the specified values of harvest in the first 3 years. In the terminal year (year 4 in this case), harvest is set equal to growth so in periods from T to infinity, the stock remains at X_T.

X_t= Initial stock size

$$X_{t+1} = X_t + G(X_t) - H_t$$
$$X_{t+2} = X_{t+1} + G(X_{t+1}) - H_{t+1}$$
$$X_{t+3} = X_{t+2} + G(X_{t+2}) - H_{t+2}$$
$$X_T = X_{t+3} + G(X_{t+3}) - G(X_{t+3})$$

The optimal harvest path can be determined with this stock projection model using the following iterative process:

Select initial stock size,
Select the time period, T
Find harvest vector {Ht,….,H_{t-1}} that will maximize NPV of harvest.

Where in all periods: $H_i \geq 0$, $X_i \geq 0$, $E(H_i) \leq Emax$. Further the NPV of harvest from T to infinity is captured as the annuitized value of the net revenue of harvesting the sustainable growth at X_T from time T to infinity. This is represented as:

$$\frac{\{P \cdot G(X_T) - E[G(X_T)] \cdot C_E\}}{(1+\delta)^t}$$

(The annuity value formula follows from: Jon M. Conrad Resource Economics. Cambridge University Press, 1999).

Finally if $X_T \neq X_{DMEY}$, increase T and start the process over.

The details and some examples are provided on the How to Get There worksheet. Please note that because the Solver tool is used on the How to Get There worksheet, it cannot be protected. Make sure you save a copy of the file is a safe place before working with this worksheet. The details of answering the Where to Go? and the How to Get There? questions are explained in sections 3 and 4 below.

2. In the worksheet "BASICS", the PEC and the EEC for this case are shown in Exercise Figure 4.1, which is a partial reproduction of Figure 4.1 from the text. The stock trajectory is also shown in the figure. The Simulation model used to derive the curve is displayed below starting at row 42. It is analogous to the models in earlier exercises and will not be explained here.

The sustainable revenue and cost curves in terms of effort and stock size are displayed in Exercise Figures 4.1A and 4.1B. The data necessary to plot them and the EEC and PEC are derived to the right of the simulation model following the procedure used in the earlier exercises. Any of the parameters in yellow boxes can be changed and the effects on the curves will be shown. Push the F9 key to recalculate.

Press ctrl + c to reset the parameters to status quo values and push F9 to recalcúlate.

3. The golden rule equation for finding the dynamically optimal stock is provided in the text.

$$G' + \frac{y_x\left[\dfrac{C'}{y_E}\right]}{\left[P - \dfrac{C'}{y_E}\right]} = \delta$$

Equation 4.16

The elements of the equation of are calculated on the Where to Go? worksheet in cells E6, F6, G6, and H6. The left hand side (LHS) of the equation is in cell D6. The discount rate, delta, from the parameter table, is placed in cell B6. A macro has been designed to find the

stock size in cell B2 that will cause the golden rule equation to hold. The levels of Y_{DMEY} and E_{DMEY} can be calculated from the value of X_{DMEY}.
To find the optimal values for other cases, change a biological, economic, or technical parameter on the Basics worksheet and push ctrl + b to run the marcro. Then push ctrl + c to reset the parameters and push F9 to recalcúlate.

4. Go to the How to Get There? Worksheet. Exercise Figure 4.5 is similar to Figure 4.5 in the text. It will be used to plot the optimal time paths that are calculated. Exercise Figure 4.5 shows the plot of the optimal harvest against time.

The stock projection model introduced in section 1 is shown below the two Figures. The initial stock size is set in cell C29, and the terminal number of years (T) specified in Cell E26. Stock changes through time through the addition of growth and the subtraction of harvest. See Column C. Growth is a function of stock size, and harvest is the control variable over the time span. See below. The effort necessary to take the scheduled harvest is calculated in column F and the value of current profits and the PV of profits in any year is calculated in columns G and H. Put the cursor over cell H29 for a comment which will explain this in more detail.

The Solver tool is used to find the harvest time path that will maximize the NPV of profits which is calculated in cell G27. Select the data file on the top toolbar and push the Solver icon on the far right. (If it is not there, use the help function to find how to install the Solver as an add-in.). The Solver is set to maximize the value in cell G27, by selecting a series of harvest is cells A29 to A79, which have been named trial harvest. For practical purposes, given the equation in column E, only harvest levels from period zero to period T-1 are considered. The iterative process is necessary because Solver will frequently be unable to find a solution if T is initially set to high.

Constraints have been specified such that all harvest levels must be greater than 1, stock size must always be greater than zero, and effort can never be greater than a specified level of maximum effort, which is set in cell I27.

The calculated values of X_{DMEY}, y_{DMEY}, and E_{DMEY} are shown in the light blue cells F22-F24. The values that are obtained from the search procedure using Solver are shown in the light green cells D22-D24. The search procedure should be continued until the values in the Search Column match or are very close to the values in the Calculation column.

Use the following procedure to solve for the optimal path when the stock needs to grow. Push ctrl and c to reset parameters. Set the interest rate in the parameter table equal to 8%, and push ctrl and b to calculate the optimal values of X, y, and E. Set the initial stock size in Cell E26 to 25,000. Because the search procedure in Solver is sometimes sensitive to the starting values, place the value of y_{DMEY}, 7,198, in all of the yellow cells in Column A. It likely will not make a difference in this case, but it could in other cases the interested reader may try.

Set years equal to 6, push the solver icon and then push solve. When solver finds a solution, click ok. The pink path shows the calculated optimal path. Note that the curve is not smooth and the calculated values and the search values do not match. Increase the number of years to 8, and run solver again. The curve is smoother and the two sets of values are closer together. Set years equal to 11 and go again. This is about as good as you can expect.

Try other discount rates and other initial stock sizes that are below 60,000. At very low initial stock sizes, the calculation process can take a long time.

To see case where the stock needs to be deceased follow these steps. Set the initial stock size in Cell E26 to 85,000. Again place the value of y_{DMEY}, 7,198, in all of the yellow cells in Column A. Set years equal to 6, push the solver icon and then push solve. When solver finds a solution, click ok. The pink path shows the calculated optimal path. We are very, very close. Set years equal to 9, and go again. In this case, an optimal path that got to the calculated value of X, Y, and E was found in two trials.

Exercise 4.2 Case 2

1. This exercise for Excel file "Chapter 4 - Case 2" is analogous to the exercise for Case 1. The first worksheet shows the parameter table and explains the PEC and the EEC curve and discusses the nature of the bioeconomic equilibrium, which is not always a certainty in this case. The next two worksheets look at the "Where to Go" and the "How to Get There" questions. The explanations are a little more complicated but the discussion will complement that in the text.

Please ensure that the updates for this file have been completed.

2. Exercise Figure 4.2 on the Basics worksheet is based on Figure 4.2 in the text. If the PEC (text equation 4.8a) and the EEC (text equation 4.9a) intersect there is a stable bioeconomic equilibrium at the higher stock size. If they do not intersect, there is no equilibrium. The price/cost ratio is an important element in the position of the EEC. Push ctrl and x to reset parameters and then change P to $15. This is an equilibrium case. Try the following combinations of initial X and E to see what happens to the trajectory. (60,000,4,000), (25,000,4,000), (100,000,6,000), (40,000, 6,000), (25,000,6,000).

Scroll over and look at the three figures to the right. Exercise Figure 4.2a shows the short run revenue and cost curves for this case. The equations for revenue and cost can be seen in text equation 4.9. The curves are independent of stock size, and so the bioeconomic equilibrium depends on economic parameters only. Exercise figures 4.2B and 4.2C show the sustainable revenue and cost curves for this case in terms of effort and stock size respectively. The sustainable curves terminate at E_{MSY}. A key factor for the possibility of a bioeconomic equilibrium is the relationship between the short run equilibrium level of E and E_{MSY}.

Push ctrl and x to reset the parameters. Then insert a price equal to $20. The EEC is so far to the right that it is off the graph in Exercise Figure 4.2 and there will never be a bioeconomic equilibirum no matter what the initial stock and effort levels. Note the differences in the remaining figures. The level of effort where short run profit equals zero is greater than E_{MEY} and the sustainable revenue curve in terms of X is everywhere above the cost curve.

Scenario 1 $E_{MEY} > E_{MSY} > E_{DMEY}$

Price equals $23 is an example.

Static E_{MEY} will cause stock to fall to zero, effort must be less than E_{MEY}.

Solution of Golden Rule Equation applies: Go to X_{DMEY} regardless of initial stock size.

Scenario 2 $E_{MSY} > E_{MEY} > E_{DMEY}$

Price equal $21 is an example.

If $X > X_{MEY}(l)$, Static $E_{MEY\ will}$ cause stock to raise or fall to $X_{MEY(h)}$ depending on exact starting point. There is no need to restrict effort below E_{MEY}. Solution of Golden Rule Equation does not apply

Go to $X_{MEY(h)}$).

If X < XMEY (l), Static E_{MEY} will cause stock to fall to zero., effort must be less than E_{MEY}.

Solution of Golden Rule Equation applies: Go to $X_{DMEY.}$

Scenario 3 $E_{MSY} > E_{DMEY} > E_{MEY}$

Price equal $18 is an example.

If $X > X_{MEY}(l)$, Static E_{MEY} will cause stock to raise or fall to $X_{MEY(h)}$ depending on exact starting point. Solution of Golden Rule Equation does not apply because $E_{MEY} < E_{DMEY}$ and it does not make sense to use increase effort up to E_{DMEY}:

Go to XMEY(h).

If X < XMEY (l), Static E_{MEY} will cause stock to fall to zero.

Solution of Golden Rule Equation does not apply because $E_{MEY} < E_{DMEY}$ and it does not make sense to use increase effort up to E_{DMEY}:

Go to $X_{MEY.(l)}$

Scenarios for Case 2

3. In the worksheet “Where to Go”, the Golden Rule equation in this case is $G'(X) = \delta$. See text equation 4.16a. The solution to this equation, X_{DMEY}, for the chosen parameters is shown

in cell G2. As explained in the text, there are certain conditions with respect to the relative sizes of E_{DMEY} and E_{MEY} and the initial stock size that will determine whether the Golden Rule solution applies. The different scenarios are explained above.

4. To conclude go to the How to Get There? worksheet. The process for solving for the optimal harvest path in Case 2 is that same as was explained for Case 1above and the details of the stock projection model will not be discussed again. The interested reader may want to look over the stock projection model and the details of the solver programming.

Use the following procedures to solve for the optimal paths for the different scenarios in this case. For scenario 1 set price equal to $23 on the basics worksheet. This is a relatively simple case because the solution of the Golden Rule equation holds. The calculated values of X_{DMEY}, y_{DMEY}, and E_{DMEY} are shown in Cells H21, H22, and H23. The optimal equilibrium will occur at the intersection of the red PEC and the maroon X_{DMEY} curve. See the analogous figure on the How to Get There worksheet. The optimal harvest trajectory will depend upon whether the Initial X is set above or below the 30,000 X_{DMEY} stock size.

Set the initial stock size to 40,000 in Cell G17. Note that this is a different relative location than in the Case 1 spreadsheet. Set the number of years to 6 in Cell C26 and start the iteration solver process. It is a good idea to set the values in the trial harvest column equal to $y_{DMEY,}$ which in this case is 6,300.. The number of years to be considered is set in cell C26. Gradually increase the number of years and run solver again and watch how the trajectory and the search values approach the calculated values. It can take quite while to approach the correct solution, but if you get in a hurry and increase the increments too rapidly solver will not find a solution and you will have to start over.

Now set the initial stock at 20,000 in Cell G17 and run the process again to watch the optimal path which will lead to X_{DMEY}. Make sure you set the trial harvest level column to y_{DMEY}.

Note that the when the initial stock is greater than X_{DMEY}, the path will lie outside the PEC. But it will lie inside the curve if the initial stock is less than X_{DMEY}. Push Ctrl +X to reset parameters.

For scenario 2, set price equal to $21.25 and push the F9 key to recalculate. The optimal values of X, y, and E are shown in Cells I21, I22, and I23. This is the "in-between" scenario that was noted but not explained in the text. A key consideration is whether the initial stock size is greater than or equal to $X_{MEY(l).}$ The two X_{MEY} curves are the blue horizontal lines. Remember the same graph showing more detail is on the Where to Go worksheet. If the initial stock size is greater $X_{MEY(l),}$ then the optimal stock size will be $X_{MEY(h)}$.

Set initial stock size to 50,000 in Cell G17 and then set the elements in the trial harvest column equal to the calculated value of y_{DMEY} which is 7,006 is this case. See the value in Cell I22. After running solver a couple of times, it can be seen that as explained in the text, the optimal path will be vertical at E_{DMEY} until the stock grows to X_{DMEY}. Now set the initial stock size equal to 80,000 in Cell G17. The same procedure will show that the optimal path will also be at E_{DMEY} and the stock will fall to X_{DMEY}.

The opposite case is where the initial stock size is less that $X_{MEY(l)}$ and in this case the optimal stock size will be X_{DMEY}. Run the cases where the initial stock sizes are 35,000 and 20,000. In the former situation, the stock size will fall and the optimal equilibrium will occur at the intersection of the red PEC and the higher blue X_{MEY} curve. In the later case it will grow and the optimal equilibrium will occur at the intersection of the red PEC and the maroon X_{DMEY} curve. . The trajectories will not be vertical lines but will be analogous to those in Case 1. It is necessary to start the 20,000 stock size case with a time period of 3. Push Ctrl +X to reset parameters.

For scenario 3 set the price equal to $18 and push the F9 key to recalculate. The optimal values of X, y, and E are shown in Cells J21, J22, and J23. This is the scenario where the golden rule solution never applies because E_{MEY} is less than E_{DMEY} and so it is not profitable to increase effort high enough to push the stock to X_{DMEY}. The solution will be at the intersection of the red PEC and on of the X_{MEY} curves depending upon the initial stock size.

If the initial X is greater that $X_{MEY(l),}$ the optimal stock size will be $X_{MEY(h).}$ For example, if the initial stock is set at 25,000, the optimal path will be vertical at E_{MEY} until stock grows to $X_{MEY(h).}$ However, if the initial stock is 95,000, the path will again be vertical at E_{MEY} but the stock will fall to $X_{MEY(h)}$. This can be verified by using the solver iteration process.

On the other hand, if the initial stock is less than $X_{MEY(l)}$, the optimal stock size will be $X_{MEY(l).}$ Consider the case where the initial stock size is 5,000. The optimal path will begin with a very low effort but it will gradually increase to E_{MEY}. The exact solution can be found using the solver process, however it is necessary to set the initial harvest column to 200 in each cell, and then start out with a single year and gradually increase a year at a time.
Push ctrl +X to reset parameters.